Mechanisms of the microwave photoconductivity in two-dimensional electron systems with mixed disorder

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We present a systematic study of the microwave-induced oscillations in the magnetoresistance of a two-dimensional electron gas for mixed disorder including both short-range and long-range components. The obtained photoconductivity tensor contains contributions of four distinct transport mechanisms. We show that the photoresponse depends crucially on the relative weight of the short-range component of disorder. Depending on the properties of disorder, the theory allows one to identify the temperature range within which the photoresponse is dominated by one of the mechanisms analyzed in the paper.

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I. INTRODUCTION

During the last decade many experimental and theoretical advances have been made in the field of nonequilibrium magnetotransport in two-dimensional (2D) electron systems. The research activity was triggered by the discovery of a number of beautiful nonequilibrium phenomena, the microwave-induced resistance oscillations (MIROs)[1,2] in the first place, governed by the ratio ω/ω_c of the radiation frequency and the cyclotron frequency $\omega_c=|e|B/mc$. Further experiments on the MIRO[3–33] led to a spectacular observation of zero resistance states[3–7] which were later explained as a result of instability leading to the formation of current domains.[34]

These discoveries stimulated an intense theoretical research[34–64] which has led to significant advances in understanding the nonequilibrium transport in high Landau levels (LLs). Initially, the MIROs were attributed to the *displacement* mechanism which accounts for spatial displacements of semiclassical electron orbits associated with the radiation-assisted scattering off disorder.[35–37] The preferred direction of such displacements along the symmetry-breaking dc field oscillates as a function of ω/ω_c due to a periodic modulation in the density of states (DOS) $\nu(\varepsilon)=\nu(\varepsilon+\hbar\omega_c)$ in a magnetic field. The displacement mechanism results in temperature independent MIRO with the phase and period as those observed in the experiments.

The dominant contribution to the MIRO was later shown to be due to the *inelastic* mechanism associated with a radiation-induced changes in the energy distribution of electrons.[38,39] The nonequilibrium part of the distribution oscillates with energy due to the oscillatory DOS and its amplitude is controlled by the inelastic relaxation time $\tau_{\rm in}\propto T^{-2}$.[39] As a consequence, the inelastic contribution to the MIRO provided an explanation of the observed strong temperature dependence of the MIRO.

Further, a recent systematic study of the photoresponse revealed two additional mechanisms of the MIRO, *quadrupole* and *photovoltaic*.[40] In the quadrupole mechanism, the microwave radiation leads to excitation of the second angular harmonic of the distribution function. The dc response in the resulting nonequilibrium state yields an oscillatory contribution to the Hall part of the photoconductivity tensor which violates Onsager symmetry.[65] In the photovoltaic mechanism, a combined action of the microwave and dc fields produces nonzero temporal harmonics of the distribution function. The response of this state to an ac electric field contributes to both the longitudinal and Hall dc photoconductivities.

The theory developed in Refs. 37, 39, and 40 assumed that the only relevant source of disorder potential is that created by remote ionized donors which are separated from the plane of 2D electrons by a wide spacer of width strongly exceeding the Fermi wavelength, $d\gg k_F^{-1}$. Such smooth disorder is characterized by scattering on small angles $\sim 1/(k_F d)$, resulting in the long transport scattering time, $\tau_{\rm tr}\sim\tau_{\rm q}(k_F d)^2\gg\tau_{\rm q}$, and the backscattering time, $\tau_\pi\propto\tau_{\rm q}\exp(2k_F d)$; here $\tau_{\rm q}$ is the total disorder-induced quantum scattering time. For a smooth disorder potential, the inelastic contribution dominates over the displacement contribution for $\tau_{\rm in}>\tau_{\rm q}$, the condition which is fulfilled in the whole temperature range where the MIROs were observed in the early experiments.[1–6] The quadrupole and photovoltaic mechanisms are the only ones yielding oscillatory corrections to the Hall part of the photoconductivity tensor, with the magnitude comparable to that of the displacement contribution to the diagonal part.

In addition to the MIRO, 2D electrons exhibit nonequilibrium magneto-oscillations when a strong current is applied to a sample in the absence of the microwave radiation.[66–70] These oscillations in the nonlinear resistivity, called Hall-induced resistance oscillations (HIROs), were attributed to the resonant backscattering off disorder between LLs tilted by the electric field.[66,71] Therefore, in contrast to the MIRO, the HIROs require a sufficient amount of short-range disorder in order to be visible in experiment. This disorder potential is created by residual charged impurities in the vicinity of the 2D electron plane. Being inevitably present in quantum Hall structures, such impurities are believed to limit the mobility of ultrahigh-mobility samples. Recent

experimental[28–30] and theoretical[63] study of the interplay of the MIRO and HIRO demonstrated that the displacement contribution to the MIRO is very sensitive to the nature of disorder. In fact, it is known that various transport phenomena in 2D electron systems crucially depend on the type of disorder, including quantum magneto-oscillations in the dc and ac transport, momentum-dependent conductivity, quasiclassical memory effects, commensurability oscillations in lateral superlattices, and interaction-induced quantum magnetoresistance (for a review, see Ref. 72). Therefore, generalization of the previous studies on the MIRO to the case of a generic-disorder potential is highly desirable.

Here we present a comprehensive analysis of the photoresponse, taking into account all four contributions to the MIRO[40] and using a generic-disorder model which is characterized by an arbitrary dependence of the elastic scattering rate

$$\tau_\theta^{-1} = \sum_{n=-\infty}^{\infty} \tau_n^{-1} e^{in\theta}, \quad \tau_n = \tau_{-n} \tag{1}$$

on the scattering angle θ. We analyze the obtained results for a realistic model of mixed disorder formed by superposition of a smooth random potential and short-range scatterers. The theory allows one to identify the temperature range where a specific mechanism dominates the photoresponse for different types of disorder.

The paper is organized as follows. In Sec. II we present a kinetic equation approach to the problem. In Sec. III we obtain the photoconductivity tensor which is analyzed in Sec. IV for different types of disorder. In Sec. V we discuss possible temperature regimes of the MIRO taking into account the interaction-induced broadening of Landau levels. Main findings are summarized in Sec. VI.

II. FORMALISM

To begin with, we briefly outline the main steps in setting up the formalism. We consider 2D electrons in a classically strong magnetic field B in the presence of a random potential, a dc electric field,

$$\boldsymbol{E}_{\rm dc} = (E_x, E_y), \tag{2}$$

and a microwave field,

$$\boldsymbol{E}_\omega(t) = E_\omega \sum_{\pm} s_\pm \, {\rm Re}[\boldsymbol{e}_\pm e^{i\omega t}], \tag{3}$$

where $\sqrt{2}\boldsymbol{e}_\pm = \boldsymbol{e}_x \pm i\boldsymbol{e}_y$ and $s_\pm$ parameterize polarization of the microwaves ($s_+^2 + s_-^2 = 1$). The main parameters in the problem are related to each other as follows:

$$\varepsilon_F \gg T, \omega, \omega_c, \tau_{\rm q}^{-1} \gg \tau_{\rm tr}^{-1},$$

where ε_F is the Fermi energy and we use $k_B = \hbar = 1$. We adopt the approach[37,39,40,63] to the problem based on the quantum Boltzmann equation (QBE) for the semiclassical distribution function $f(\varepsilon, \varphi, t)$ at higher LLs,

$$(\partial_t + \omega_c \partial_\varphi) f + \tau_{\rm in}^{-1}(\langle f \rangle - f_T) = {\rm St}\{f\}. \tag{4}$$

In the inelastic collision integral $\propto \tau_{\rm in}^{-1}$, $f_T(\varepsilon)$ is the Fermi-Dirac distribution. The angular brackets $\langle \cdots \rangle$ denote averaging over the direction φ of the kinematic momentum.

The QBE [Eq. (4)] allows us to treat the interplay of the disorder, the Landau quantization, and the external fields, which are all included into the impurity collision integral ${\rm St}\{f\}$. The field-dependent collision integral appears as a consequence of transition to a moving coordinate frame, $\boldsymbol{r} \to \boldsymbol{r} - \boldsymbol{\zeta}_t$, where

$$\partial_t \boldsymbol{\zeta}_t = \left(\frac{\partial_t - \omega_c \hat{\varepsilon}}{\partial_t^2 + \omega_c^2} \right) \frac{e}{m} [\boldsymbol{E}_{\rm dc} + \boldsymbol{E}_\omega(t)] \tag{5}$$

and $\hat{\varepsilon}_{xy} = -\hat{\varepsilon}_{yx} = 1$. In the new frame, the electric field is absent, but the impurities are moving and can change the energy of electrons. The effects of generic disorder and external fields are encoded in the integral operator,[63]

$$\hat{\mathcal{K}}_{t_2,t_1;\varphi}\{F(\varphi)\} = \int \frac{d\varphi'}{2\pi} \frac{e^{ip_F(\boldsymbol{n}_\varphi - \boldsymbol{n}_{\varphi'})(\boldsymbol{\zeta}_{t_2} - \boldsymbol{\zeta}_{t_1})}}{\tau_{\varphi-\varphi'}} F(\varphi'), \tag{6}$$

where $p_F = mv_F$ is the Fermi momentum, the unit vector $\boldsymbol{n}_\varphi = (\cos\varphi, \sin\varphi)$, and $F(\varphi)$ is an arbitrary function. In terms of $\hat{\mathcal{K}}$, the scattering integral in the time representation is given by the expression

$$\begin{aligned} {\rm St}\{f\}_{t_2,t_1} = \int dt_3 [& \hat{\mathcal{K}}_{t_2,t_1}\{g^R_{t_2-t_3} f_{t_3,t_1} - f_{t_2,t_3} g^A_{t_3-t_1}\} \\ & - f_{t_3,t_1} \hat{\mathcal{K}}_{t_2,t_3}\{g^R_{t_2-t_3}\} + f_{t_2,t_3} \hat{\mathcal{K}}_{t_3,t_1}\{g^A_{t_3-t_1}\}]. \end{aligned} \tag{7}$$

We are going to explore the regime of overlapping LLs, $\omega_c \tau_{\rm q} \ll 1$, when the spectral functions

$$g^R_{t_2-t_1} = [g^A_{t_2-t_1}]^\dagger = \delta(t_2 - t_1)/2 - \lambda \delta(t_2 - t_1 - t_B) \tag{8}$$

depend on a single parameter

$$\lambda = \exp(-\pi/\omega_c \tau_{\rm q}) \ll 1 \tag{9}$$

and are insensitive to the external electric fields. The corresponding DOS is

$$\nu(\varepsilon) = \nu_0 [1 - 2\lambda \cos(\varepsilon t_B)], \tag{10}$$

where $t_B = 2\pi/|\omega_c|$ is the cyclotron period and $\nu_0 = m/2\pi$ is the DOS at $B=0$ per spin degree of freedom.

The Wigner transform (Fourier transform with respect to $t_- = t_2 - t_1$) of the scattering integral [Eq. (7)] defines the QBE for the distribution function $f(\varepsilon, \varphi, t)$, which can be approximated by

$$f = f_T + (\partial_\varepsilon f_T)\{\phi_0^D - 2\lambda \, {\rm Re}[\phi_1 \exp(i\varepsilon t_B)] + 2\lambda^2 \phi_0^{(2)}\}, \tag{11}$$

where

$$\phi_0^D(\varphi) = ev_F \boldsymbol{E}_{\rm dc} \cdot \boldsymbol{n}_\varphi / \omega_c^2 \tau_{\rm tr} \tag{12}$$

is the classical part leading to the Drude expression for the current [Eq. (16)] and $t = (t_2 + t_1)/2$. By the symmetry of the kernel [Eq. (6)],

$$\phi_\perp \equiv {\rm Im}\, \phi_1(\varphi, t) \tag{13}$$

includes even angular harmonics of the distribution function and is governed by the real part of the kernel [Eq. (6)],

$$(\partial_t + \omega_c\partial_\varphi)\phi_\perp + \tau_{\rm in}^{-1}\langle\phi_\perp\rangle = \left(\tfrac{1}{2}\partial_t - \partial_{t_-} + \phi_\perp\right)\mathrm{Re}\,\hat{\mathcal{K}}_{t_2,t_1}\{1\}\Big|_{t_2=t_1+t_B}. \quad (14)$$

The dc electric current,

$$\boldsymbol{j} = 2ev_F\int d\varepsilon\,\nu(\varepsilon)\overline{\langle \boldsymbol{n}_\varphi f(\varepsilon,\varphi,t)\rangle} - 2e\nu_0\varepsilon_F\overline{\partial_t\boldsymbol{\zeta}(t)}, \quad (15)$$

is determined by the first angular harmonic of the distribution function [Eq. (11)], which is present in Re ϕ_1, $\phi_0^{(D)}$, and $\phi_0^{(2)}$. The bar denotes time averaging over the period of the microwave field in the steady state.

Our aim is to calculate the current [Eq. (15)] for generic disorder [Eq. (1)] to order $E_\omega^2 E_{\rm dc}$ for temperatures $T \gg \omega, \omega_c$. This regime is the most relevant to experiments on the MIRO and allows for a reliable comparison between the theory and experiment. In the absence of quantum corrections ($\lambda=0$) the current is given by the Drude formula,

$$\boldsymbol{j}^D = 2\sigma_D(1-\omega_c\tau_{\rm tr}\hat{\varepsilon})\boldsymbol{E}_{\rm dc}, \quad (16)$$

where $\sigma_D = e^2\nu_0 v_F^2/2\omega_c^2\tau_{\rm tr}$. Neither strong dc field nor microwave modifies this result as long as $\lambda=0$ and the energy dependence of elastic scattering (weak at $\omega_c \ll \varepsilon_F$) is neglected.[48] All quantum corrections to $\nu(\varepsilon)f(\varepsilon)$ of first order in λ oscillate with energy and are exponentially suppressed at $2\pi^2 T/\omega_c \gg 1$ similar to Shubnikov–de Haas (SdH) oscillations. The leading quantum corrections which survive the temperature smearing at $T \gg \omega_c$ are of order λ^2 and can be written using Eqs. (11) and (15) as

$$\mathbf{j} = \mathbf{j}^D - 4\lambda^2 ev_F\nu_0(\mathrm{Re}\,J, \mathrm{Im}\,J)^T \quad (17)$$

with

$$J = \left\langle \frac{e^{i\varphi}}{\omega_c}\left(\partial_{t_-}\overline{\hat{\mathcal{K}}_{t_2,t_1}\{1\}} - \overline{\phi_\perp\hat{\mathcal{K}}_{t_2,t_1}\{1\}} - \overline{\hat{\mathcal{K}}_{t_2,t_1}\{\phi_\perp\}}\right)\right\rangle_{t_-=t_B}. \quad (18)$$

III. PHOTOCONDUCTIVITY TENSOR

We show that the current J [Eq. (18)] depends in an essential way on the properties of disorder, specifically on whether it scatters isotropically (short-range disorder) or primarily on small angles (long-range disorder). A straightforward calculation of J to the first order in $E_{\rm dc}$ and to the second order in E_ω using the solution $\phi_\perp$ of Eq. (14) gives the following expression for the current [Eq. (17)]:

$$\boldsymbol{j} = \boldsymbol{j}_D + 4\sigma_D\lambda^2\boldsymbol{E}_{\rm dc} + \hat{\sigma}^{(\rm ph)}\boldsymbol{E}_{\rm dc}, \quad (19)$$

$$\hat{\sigma}^{(\rm ph)} = -4\sigma_D\begin{pmatrix} d_s + d_a & h_s + h_a \\ h_s - h_a & d_s - d_a\end{pmatrix}. \quad (20)$$

The terms of different symmetries $d_{s,a}$ and $h_{s,a}$ in the photoconductivity tensor $\hat{\sigma}^{(\rm ph)}$ are given below for arbitrary strength of the short-range and smooth components of disorder. The diagonal part of the photoconductivity tensor consists of the isotropic, d_s, and anisotropic, d_a, components, where

$$d_s = d_s^{(A)} + d_s^{(B)} + d_s^{(D)}, \quad (21a)$$

$$d_s^{(A)} = \lambda^2[\sin^2 w + w\sin 2w]\frac{\tau_{\rm tr}}{2\tau_*}\sum_\pm \mathcal{E}_\pm^2, \quad (21b)$$

$$d_s^{(B)} = \lambda^2[w\sin 2w]\frac{2\tau_{\rm in}}{\tau_{\rm tr}}\sum_\pm \mathcal{E}_\pm^2, \quad (21c)$$

$$d_s^{(D)} = -\lambda^2[w\sin^2 w]\sum_\pm \mathcal{E}_\pm^2\left(\frac{2}{\omega\tau_{\rm tr}} + \frac{2\tau_{\rm tr}\tilde{\tau}^{-2}}{\omega \pm 2\omega_c}\right) \quad (21d)$$

and

$$d_a = d_a^{(A)} + d_a^{(D)}, \quad (21e)$$

$$d_a^{(A)} = -\lambda^2[\sin^2 w + w\sin 2w]\frac{\tau_{\rm tr}}{2\tau_*}\mathcal{E}_+\mathcal{E}_-, \quad (21f)$$

$$d_a^{(D)} = \lambda^2[w\sin^2 w]\frac{4}{\omega\tau_{\rm tr}}\mathcal{E}_+\mathcal{E}_-. \quad (21g)$$

In the off-diagonal part, h_a represents the microwave-induced correction to the Hall conductivity, while h_s is the anisotropic contribution to the dissipative conductivity (violating the Onsager symmetry),[40,65]

$$h_s = h_s^{(C)} = -\lambda^2[w\sin 2w](\tau_{\rm tr}/\omega_c\tilde{\tau}^2)\mathcal{E}_+\mathcal{E}_-, \quad (22a)$$

$$h_a = h_a^{(D)} = -\lambda^2[2\sin^2 w + w\sin 2w] \times \sum_\pm \pm\mathcal{E}_\pm^2\left(\frac{1}{\omega\tau_{\rm tr}} + \frac{\tau_{\rm tr}\tilde{\tau}^{-2}}{\omega \pm 2\omega_c}\right). \quad (22b)$$

All microwave-induced corrections [Eqs. (21) and (22)] show oscillations with the ratio

$$w = \frac{\omega t_B}{2} = \frac{\pi\omega}{|\omega_c|} \quad (23)$$

and are of the second order in the microwave field E_ω measured in dimensionless units,

$$\mathcal{E}_\pm = s_\pm\frac{ev_F E_\omega}{\omega(\omega \pm \omega_c)}. \quad (24)$$

In Eqs. (21) and (22), the superscripts denote the contributions from the displacement (A), inelastic (B), quadrupole (C), and photovoltaic (D) mechanisms.[40] The displacement contribution (A) comes from the first term in Eq. (18). In the absence of the microwave radiation, $E_\omega=0$, this term produces the quantum correction to the Drude conductivity given by the second term in Eq. (19). At order $E_{\rm dc}E_\omega^2$, the first term in Eq. (18) accounts for the microwave-assisted disorder scattering in the presence of the symmetry-breaking dc field. Therefore, in the displacement mechanism the radiation directly affects the first angular harmonic of the distribution function and thus the dc [see Eqs. (21b) and (21f)]. Mechanisms (B)-(D) are related to the microwave excitation of even angular harmonics of the distribution function governed

by Eq. (14). Their contribution to the current appears at the same order $E_{\rm dc}E_\omega^2$ and is described by two last terms in Eq. (18). The microwave-induced changes in the time-independent zero angular harmonics of the distribution function are controlled by the inelastic relaxation and lead to the inelastic contribution (B) [Eq. (21c)]. In the quadrupole mechanism (C), the microwave radiation excites the second angular harmonic of the distribution function, $\phi_\perp^{(C)} \propto E_\omega^2 \cos 2\phi$, which also contributes to the dc at order $E_{\rm dc}E_\omega^2$ after substitution into Eq. (18) with $\mathcal{K} \propto E_{\rm dc}$ [see Eq. (22a)]. In the photovoltaic mechanism (D), a combined action of the microwave and dc fields in Eq. (14) produces both zero and second angular harmonics of the distribution function $\phi_\perp$ which oscillate in time with the microwave frequency. The ac response in the resulting state [Eq. (18) with $\mathcal{K} \propto E_\omega$ and $\phi_\perp \propto E_\omega E_{\rm dc}$] contributes to the longitudinal and Hall parts of $\hat{\sigma}^{\rm (ph)}$ [see Eqs. (21d), (21g), and (22b)].

The photoconductivity tensor [Eq. (20)] depends on the inelastic scattering rate $\tau_{\rm in}^{-1}$ which enters the inelastic contribution [Eq. (21c)] and on four different disorder scattering rates. The latter are expressed in terms of the angular harmonics τ_n of the disorder scattering rate [Eq. (1)] as

$$\frac{1}{\tau_{\rm q}} = \frac{1}{\tau_0}, \tag{25a}$$

$$\frac{1}{\tau_{\rm tr}} = \frac{1}{\tau_0} - \frac{1}{\tau_1}, \tag{25b}$$

$$\frac{1}{\tau_*} = \frac{3}{\tau_0} - \frac{4}{\tau_1} + \frac{1}{\tau_2}, \tag{25c}$$

$$\frac{1}{\tilde{\tau}} = \frac{1}{\tau_1} - \frac{1}{2\tau_0} - \frac{1}{2\tau_2}. \tag{25d}$$

We emphasize that as compared to the case of smooth disorder, where the rates $\tau_{\rm q}^{-1}$ and $\tau_{\rm tr}^{-1}$ fully parametrize the photoconductivity,[40] two additional rates τ_*^{-1} and $\tilde{\tau}^{-1}$ are required in general situation. The rate τ_*^{-1} controls the magnitude of the displacement contributions [Eqs. (21b) and (21f)], reproducing the result in Ref. 63. Note that, in view of $\sigma_D \propto \tau_{\rm tr}^{-1}$, the product $\sigma_D d_s^{(A)} \propto \tau_*^{-1}$ entering Eq. (20) is actually independent of $\tau_{\rm tr}$. The rate $\tilde{\tau}^{-1}$ enters the quadrupole and photovoltaic contributions [Eqs. (21d), (21g), (22a), and (22b)]. The relation between the scattering rates [Eq. (24)] strongly depends on the type of disorder potential in the plane of 2D electrons (short range vs long range). We discuss this dependence in Sec. IV for a realistic model of the disorder.

IV. DEPENDENCE OF MIRO ON TYPE OF DISORDER

In this section we study the relative magnitude of the contributions of mechanisms (A)-(D) for different types of disorder. Using Eqs. (21), (22), and (25) we demonstrate that the MIRO amplitude is very sensitive to the details of the disorder potential and may provide a valuable information about the disorder which cannot be extracted from the conventional transport measurements of the mobility and the SdH oscillations.[73]

A. Mixed-disorder model

The major source of elastic scattering in high-mobility Hall structures is a smooth random potential created by remote donors that are separated by a large spacer of width $d \gg k_F^{-1}$ from the two-dimensional electron gas plane. The disorder model which takes into account only such remote charged impurities was used in a number of theoretical works on the MIRO[37,39,40] since it allows one to account for the experimentally relevant small-angle scattering condition $\tau_{\rm q} \ll \tau_{\rm tr}$ and to consider in an unambiguous way the region of magnetic fields $\tau_{\rm tr}^{-1} \ll \omega_c \lesssim \tau_{\rm q}^{-1}$. Here we implement a more realistic[74] "two-component" (or "mixed") model of disorder[75] which includes strong short-range scatterers in addition to the smooth potential.

In terms of the angular harmonics of the elastic scattering rate [Eq. (1)], the mixed-disorder model is formulated as[63,71]

$$\frac{1}{\tau_n} = \frac{\delta_{n,0}}{\tau_{\rm sh}} + \frac{1}{\tau_{\rm sm}} \frac{1}{1+\chi n^2}. \tag{26}$$

Here the isotropic scattering rate $\tau_{\rm sh}^{-1}$ characterizes the short-range disorder created by residual impurities located at or near the interface, while the smooth part is defined by the total (quantum) rate $\tau_{\rm sm}^{-1}$ and by the parameter $\chi = (k_F d)^{-2} \ll 1$ giving a characteristic scattering angle $\theta \sim \sqrt{\chi} \ll 1$.

Since in high-mobility samples $\tau_{\rm tr} \gg \tau_{\rm q}$, we imply that $\tau_{\rm sh}^{-1} \ll \tau_{\rm sm}^{-1}$ and the smooth component dominates in the quantum relaxation rate,

$$\frac{1}{\tau_{\rm q}} = \frac{1}{\tau_0} = \frac{1}{\tau_{\rm sh}} + \frac{1}{\tau_{\rm sm}}. \tag{27}$$

At the same time, the relative weight of the sharp and smooth components in the transport relaxation rate

$$\frac{1}{\tau_{\rm tr}} = \frac{1}{\tau_{\rm sh}} + \frac{1}{\tau_{\rm sm}} \frac{\chi}{1+\chi} \tag{28}$$

can be arbitrary.

B. Photoconductivity for smooth disorder

It is useful to recall the results in Ref. 40 for the case of smooth disorder and to see how they are obtained from Eqs. (21), (22), and (25). Setting $\tau_{\rm sh}^{-1} = 0$ (no short-range disorder), the scattering rates that enter Eq. (24) can be rewritten as

$$\frac{1}{\tau_{\rm q}} = \frac{1}{\tau_{\rm sm}}, \tag{29a}$$

$$\frac{1}{\tau_{\rm tr}} = \frac{\chi}{\tau_{\rm q}}, \tag{29b}$$

$$\frac{1}{\tau_*} = \frac{12\chi^2}{\tau_{\rm q}}, \tag{29c}$$

$$\frac{1}{\tilde{\tau}} = \frac{1}{\tau_{\rm tr}}. \tag{29d}$$

As a result, in the case of smooth disorder the ratio of the inelastic contribution [Eq. (21c)] to the displacement contribution [Eq. (21b)] is

$$\frac{d_s^{(B)}}{d_s^{(A)}} = \frac{4\tau_{\rm in}\tau_*}{\tau_{\rm tr}^2} = \frac{\tau_{\rm in}}{3\tau_{\rm q}}, \quad \tau_{\rm sh}^{-1} = 0. \tag{30}$$

This relation follows from Eqs. (21b) and (21c) if one disregards the term $\propto \sin^2 w$ which is small at $w = \pi\omega/\omega_c \gg 1$.

For typical conditions of the MIRO experiments, the parameters entering Eqs. (21), (22), and (29) can be estimated as $\omega_c \sim \omega \sim \tau_{\rm q}^{-1}$ and $\chi = \tau_{\rm q}/\tau_{\rm tr} \simeq 10^{-2}$. Using the estimate $\tau_{\rm in} \sim \varepsilon_F/T^2$ for the inelastic scattering time, we obtain $\tau_{\rm in} \sim \tau_{\rm tr}$ at $T = 1$ K. Therefore, the ratio [Eq. (30)] remains small up to $T \sim 10K$ (where the MIROs are strongly suppressed). The magnitude of the oscillations produced by the quadrupole and photovoltaic mechanisms is of the order of the displacement contribution, as they contain a small prefactor $(\omega\tau_{\rm tr})^{-1} \sim (\omega_c\tau_{\rm tr})^{-1} \sim \tau_{\rm q}/\tau_{\rm tr}$.

We come to the conclusion that, in the case of smooth disorder, the photoconductivity tensor is dominated by inelastic mechanism (B). It produces the isotropic diagonal contribution [Eq. (21c)] proportional to $\tau_{\rm in} \sim \varepsilon_F/T^2$. Other mechanisms produce weak anisotropy of the photoconductivity tensor [Eqs. (21f), (21g), and (22a)] and govern the Hall part of the photoconductivity [Eq. (22b)].

C. Photoconductivity for mixed disorder

Now we return to the mixed-disorder model [Eq. (26)] with nonzero $\tau_{\rm sh}^{-1}$ and analyze the dependence of various contributions to the photoconductivity tensor [Eq. (20)] on the weight $x = \tau_{\rm tr}/\tau_{\rm sh}$ of the sharp component of disorder in the transport relaxation rate [Eq. (28)]. In this analysis, we fix both $\tau_{\rm tr}$ [Eq. (28)] and $\tau_{\rm q}$ [Eq. (27)] and use as a parameter the ratio $\alpha = \tau_{\rm tr}/\tau_{\rm q}$ which can be extracted from the experiment.[73]

Inspection of Eqs. (21) and (22) shows that only the inelastic contribution [Eq. (21c)] is independent of the type of disorder at fixed $\tau_{\rm tr}$ and $\tau_{\rm q}$,

$$d_s^{(B)} \propto \tau_{\rm in}/\tau_{\rm tr} = {\rm const}(x). \tag{31}$$

The displacement contributions $d_s^{(A)}$ and $d_a^{(A)}$ [Eqs. (21b) and (21f)] are proportional to the ratio

$$\frac{\tau_{\rm tr}}{\tau_*} = 3\frac{\alpha x - 5x + 4}{\alpha - 4x + 3}, \tag{32}$$

which is illustrated in Fig. 1(a) for several values of the parameter $\alpha = \tau_{\rm tr}/\tau_{\rm q}$. Since in high-mobility structures the parameter α is always large, Eq. (32) demonstrates that the displacement contribution is parametrically enhanced in the case of short-range disorder, $\tau_*|_{x=1}/\tau_*|_{x=0} = 4\alpha \gg 1$.

For the ratio $(\tau_{\rm tr}/\tilde{\tau})^2$ which enters the photovoltaic and quadrupole contributions [Eqs. (21d)–(21g), (22a), and (22b)], we obtain the dependence illustrated in Fig. 1(b),

$$\frac{\tau_{\rm tr}^2}{\tilde{\tau}^2} = \left(\frac{2\alpha - 3\alpha x + 7x - 6}{2\alpha - 8x + 6}\right)^2. \tag{33}$$

Unlike the displacement contribution [Fig. 1(a)], the photovoltaic and quadrupole contributions are at their maximum in

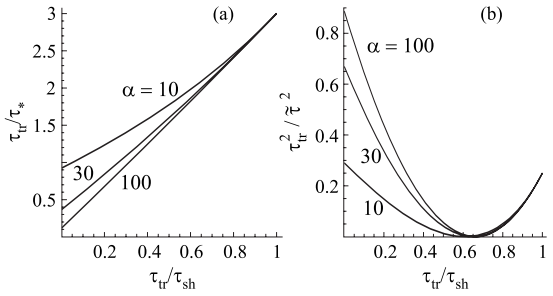

FIG. 1. Dependence of the relaxation rates (a) τ_*^{-1} and (b) $\tau_{\rm tr}/\tilde{\tau}^2$ on the type of disorder (parametrized by the weight $\tau_{\rm tr}/\tau_{\rm sh}$ of the short-range component of disorder in the transport relaxation rate) for fixed $\tau_{\rm tr}$ and $\tau_{\rm q}$ for three values of $\alpha = 100, 30, 10$.

the absence of the sharp component, $x = 0$, and do not change parametrically in the opposite limit: $\tilde{\tau}^{-2}|_{x=1}/\tilde{\tau}^{-2}|_{x=0} = 1/4$ for $\alpha \gg 1$. However, the two contributions change significantly as a function of x between $x = 0$ and $x = 1$ since at $x = (2\alpha - 6)/(3\alpha - 7)$ the effective rate $\tilde{\tau}^{-1}$ changes sign.

Summarizing, for a sufficiently large concentration of short-range scatterers, i.e., $x \sim 1$ and $\tau_{\rm sh} \sim \tau_{\rm tr}$, the ratio $\tau_{\rm tr}/\tau_*$ controlling the displacement contribution [Eqs. (21b) and (21f)] is of order unity [see Eq. (32) and Fig. 1(a)]. For typical experimental parameters, the ratio $\tau_{\rm in}/\tau_{\rm tr}$ entering the inelastic contribution [Eq. (21c)] is of order unity at $T \sim 1$ K (see Sec. IV B). Therefore, unlike the case of smooth disorder discussed in Sec. IV B, in the present scenario of strong short-range disorder both mechanisms produce comparable contributions to the diagonal isotropic component d_s of the photoconductivity [Eq. (20)].

The displacement contribution to MIRO was recently considered also for the opposite limit of well separated LLs ($\omega_c\tau_{\rm q} \gg 1$) using a different version of the two-component disorder model.[52] A similar conclusion was drawn that the displacement contribution may exceed the inelastic contribution when the transport relaxation time $\tau_{\rm tr}$ (a) exceeds $\tau_{\rm in}$ and (b) is dominated by the short-range component of the disorder potential. The results for the displacement mechanism [Eqs. (21b) and (21f)] (obtained in Ref. 63 for the regime of overlapping LLs) can be easily generalized to the case of separated LLs. The corresponding general expressions in terms of the DOS (valid in both limits $\omega_c\tau_{\rm q} \gg 1$ and $\omega_c\tau_{\rm q} \ll 1$, as well as in the crossover region $\omega_c\tau_{\rm q} \sim 1$) can be found, for instance, in Ref. 64.

In contrast to the displacement contribution, the photovoltaic and quadrupole contributions $d_{s,a}^{(D)}$, h_s, and h_a at large concentration of short-range scatterers still contain the small factor $(\omega\tau_{\rm tr})^{-1} \sim (\omega_c\tau_{\rm tr})^{-1}$ which is of order $\tau_{\rm q}/\tau_{\rm tr}$ for $\omega_c\tau_{\rm q} \sim 1$. Consequently, the contributions $d_{s,a}^{(D)}$ to the diagonal part of the photoconductivity tensor can be neglected. The terms h_s and h_a are also small; however, they remain the only microwave-induced contributions to the off-diagonal part of the photoconductivity tensor and, therefore, are important.

Since the displacement and inelastic mechanisms are equally important in the diagonal photoresponse in the case of strong short-range disorder, $x \sim 1$, it is natural to discuss possible ways to separate their contributions experimentally. As we argue below, the difference in the temperature depen-

dence of the two contributions can be used to extract the contributions separately. In Sec. V the temperature dependence of various contributions is analyzed in detail.

V. TEMPERATURE DEPENDENCE OF MIRO FOR SMOOTH AND MIXED DISORDER

A. Temperature dependence of MIRO for disorder-broadened Landau levels

In the photoconductivity tensor, as given by Eqs. (20)–(22), the only T-dependent quantity is the inelastic scattering time $\tau_{\rm in}$ entering the inelastic contribution [Eq. (21c)]. According to the analysis in Ref. 39, the inelastic relaxation of the ε oscillations in the distribution function is dominated by electron-electron (e-e) collisions. Under conditions of the MIRO experiments, the e-e collision rate for an electron at energy ε counted from the Fermi level ε_F is

$$\frac{1}{\tau_{\text{e-e}}(\varepsilon,T)} = \frac{\varepsilon^2+\pi^2T^2}{4\pi\varepsilon_F}\ln\frac{\kappa v_F}{\max\{T,\omega_c(\omega_c\tau_{\rm tr})^{1/2}\}}, \tag{34}$$

with the inverse screening length $\kappa=4\pi e^2\nu_0$.[39] The effective inelastic scattering time entering Eq. (21c) is given by the thermal average of the out-scattering time,

$$\tau_{\rm in} = \{\tau_{\text{e-e}}(\varepsilon,T)\}_T \simeq 0.822\tau_{\text{e-e}}(0,T), \tag{35}$$

where the notation

$$\{F_\varepsilon\}_T \equiv -\int d\varepsilon F_\varepsilon \partial_\varepsilon f_T(\varepsilon) \tag{36}$$

has been introduced.

The above results are applicable as long as the disorder alone controls both the momentum relaxation and the energy dependence of the DOS. Namely, when the contributions of the inelastic scattering mechanisms to the momentum relaxation rate, $\tau_{\rm tr}^{-1}$, and the quantum scattering rate, $\tau_{\rm q}^{-1}$, are negligible. This is usually the case at $T\sim 1$ K, which is a typical temperature in MIRO experiments.

B. Effect of interactions on the density of states

At higher temperatures, the momentum relaxation is modified by the phonon effects, while the DOS is influenced by both the e-e and electron-phonon interactions.[27,32,42,50] In the limit of overlapping LLs, the effect of the interactions on the DOS amounts to substituting λ in Eqs. (11) and (15) with

$$\tilde{\lambda}(\varepsilon,T) = \exp\left(-\frac{\tau_0^{-1}+\tau_{\text{e-e}}^{-1}(\varepsilon,T)+\tau_{\text{e-ph}}^{-1}(\varepsilon,T)}{\omega_c/\pi}\right). \tag{37}$$

The phonon effects will be considered elsewhere.[76] Here we discuss the modification of the temperature dependence of the photoconductivity tensor [Eqs. (20), (21a)–(21g), (22a), and (22b)] caused by the effect of the e-e interaction on the DOS (in recent experiments[27,32] the e-e interaction dominates the effect on the DOS). As shown below, the magnitude of the above effect is controlled by the parameter

$$\gamma(T) = \frac{2\pi}{\omega_c\tau_{\text{e-e}}(0,T)}. \tag{38}$$

C. Interaction-induced exponential T decay of the MIRO

In order to account for the interaction-induced variation [Eq. (37)] of the amplitude of the DOS oscillations (which is slow on the scale $\omega_c \ll T$), we make the replacement

$$\lambda^2 \equiv \{\lambda^2\}_T \to f_1(T) = \{\tilde{\lambda}^2(\varepsilon,T)\}_T \tag{39}$$

everywhere in Eqs. (21) and (22) except for the inelastic contribution $d_s^{(B)}$ [Eq. (21c)]. In the latter case, the replacement is [see Eq. (35)]

$$\lambda^2\frac{\tau_{\rm in}}{\tau_{\rm tr}} \to f_2(T) = \left\{\tilde{\lambda}^2(\varepsilon,T)\frac{\tau_{\text{e-e}}(\varepsilon,T)}{\tau_{\rm tr}}\right\}_T. \tag{40}$$

At $\gamma \ll 1$, the interaction-induced modification of the DOS is small, yielding

$$f_1 = \lambda^2(1-\gamma/3), \quad \gamma \ll 1, \tag{41}$$

$$f_2 = \lambda^2\left(\frac{\tau_{\rm in}}{\tau_{\rm tr}} - \frac{2\pi}{\omega_c\tau_{\rm tr}}\right), \quad \gamma \ll 1. \tag{42}$$

The correction to the inelastic contribution $d_s^{(B)}$ is temperature independent, while all other contributions acquire a weak temperature dependence through $\gamma \propto T^2$.

In the opposite limit $\gamma \gg 1$,

$$f_1 = \lambda^2\frac{\pi^{3/2}e^{-\gamma}}{4\gamma^{1/2}}, \quad \gamma \gg 1, \tag{43}$$

$$f_2 = \lambda^2\frac{\pi^{5/2}e^{-\gamma}}{2\gamma^{3/2}\omega_c\tau_{\rm tr}}, \quad \gamma \gg 1. \tag{44}$$

In this high temperature limit, all the quantum effects $\propto\lambda^2$ that survive the thermal averaging at $2\pi^2T/\omega_c \gg 1$ are exponentially suppressed by the inelastic e-e scattering.

Interestingly, the SdH oscillations behave just in the opposite way: they are exponentially suppressed by thermal smearing at $2\pi^2T/\omega_c \gg 1$ but they are not influenced[77–79] by the interactions that lead to the dependence of $\tau_{\text{e-e}}^{-1}$ on ε in the form $\tau_{\text{e-e}}^{-1} \propto \varepsilon^2+\pi^2T^2$. Indeed, for such a specific form of $\tau_{\text{e-e}}^{-1}$ entering Eq. (37), the thermal average produces the unperturbed result, $\{\tilde{\lambda}(\varepsilon,T)\cos(2\pi\varepsilon/\omega_c)\}_T \to \lambda\{\cos(2\pi\varepsilon/\omega_c)\}_T$ for $T \gg \omega_c$, because in this case the energy-dependent term in $\tau_{\text{e-e}}^{-1}$ effectively shrinks the range of energy integration and increases $\{\cos(2\pi\varepsilon/\omega_c)\}_T$ to compensate $\exp[-\pi/\omega_c\tau_{\text{e-e}}(0,T)]$.

D. Temperature regimes of the MIRO for smooth and sharp disorder

It follows from the above arguments that different temperature dependences of magnetoresistance are expected for samples with smooth and sharp disorder. In the former case $\tau_{\rm tr}/\tau_* \ll 1$, and the inelastic contribution [Eq. (21c)] is bigger than all other contributions by a factor of $\tau_{\rm in}/\tau_{\rm q}$ [see Eq. (30)]. This ratio is estimated as $\tau_{\rm in}/\tau_{\rm q} \sim 10-10^2$ at $T=1$ K, making the inelastic mechanism dominant in the whole range of temperatures where the MIROs are observed. The temperature dependence of the diagonal isotropic component d_s

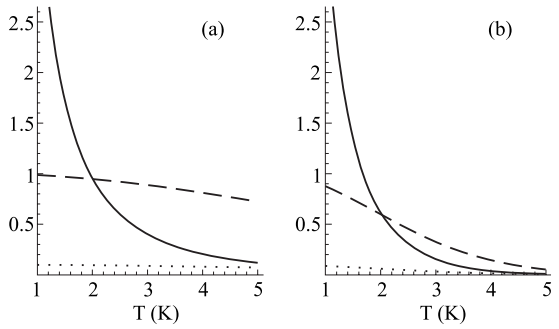

FIG. 2. *Solid lines:* T dependence of the magnitude of the inelastic contribution [Eq. (21c)], $2\{\tau_{e\text{-}e}(\varepsilon,T)\tilde{\lambda}^2(\varepsilon,T)\}_T/\tau_{tr}\lambda^2$, for $\tau_{in}|_{T=1\ \mathrm{K}}=2\tau_{tr}$. *Dashed lines:* T dependence of the magnitude of the displacement contribution [Eq. (21b)], $\tau_{tr}\{\tilde{\lambda}^2(\varepsilon,T)\}_T/2\tau_*\lambda^2$, for $\tau_{tr}/2\tau_*=1$ (which corresponds to $\tau_{tr}/\tau_{sh}=0.6$–0.7 in Fig. 1). *Dotted lines:* T dependence of the magnitude of the displacement contribution [Eq. (21b)], $\tau_{tr}\{\tilde{\lambda}^2(\varepsilon,T)\}_T/2\tau_*\lambda^2$, for $\tau_{tr}/2\tau_*=0.1$ [which corresponds to smooth disorder, for instance, $\tau_{tr}/\tau_{sh}=0$ and $\tau_{tr}/\tau_q\simeq 50$ in Fig. 1; see Eq. (32)]. *Left and right panels* correspond to (a) weak effect of interactions on the DOS at $T=1$ K, $\gamma_1\equiv 2\pi/\omega_c\tau_{e\text{-}e}(\varepsilon_F,1\ \mathrm{K})=0.01$, and to (b) strong effect, $\gamma_1=0.1$.

of the photoconductivity tensor [Eq. (20)] in the case of smooth disorder is then given by the function f_2 [Eqs. (40), (42), and (44)], which is shown in Fig. 2 by the solid line. Specifically, the T^{-2} behavior [Eq. (42)] at low T crosses over to the exponential decay [Eq. (44)] at high T.

The situation changes in the case of strong short-range component of disorder, $\tau_{tr}/\tau_{sh}\sim 1$. Since $\tau_{tr}/\tau_*\sim 1$ in this case, the displacement contribution [Eq. (21b)] to the diagonal isotropic component d_s of the photoconductivity tensor [Eq. (20)] starts to dominate over the inelastic contribution [Eq. (21c)] at a certain characteristic temperature T_1 at which the MIROs are still strong. The temperature T_1 in kelvin is given by the relation

$$T_1 = 1\ \mathrm{K}\times\frac{2}{\tau_{tr}}\sqrt{\tau_*\tau_{in}(1\ \mathrm{K})} \tag{45}$$

[see Eqs. (30), (34), and (35)]. Experimentally, T_1 is of order 1 K for the case of strong sharp component of disorder since both τ_*/τ_{tr} and $\tau_{in}(1\ \mathrm{K})/\tau_{tr}$ are of order unity. Another characteristic temperature T_2 is given by the relation $\gamma(T_2)=1$ [see Eqs. (35) and (38)],

$$T_2 = \frac{1\ \mathrm{K}}{\gamma^{1/2}(1\ \mathrm{K})}\simeq 1\ \mathrm{K}\times 0.44\sqrt{\omega_c\tau_{in}(1\ \mathrm{K})}. \tag{46}$$

It separates the temperature region $T\gtrsim T_2$ where the LL broadening induced by the e-e interaction becomes essential.

In the case $T_1\ll T_2$ the diagonal symmetric part d_s of the phoconductivity [Eq. (20)] has different temperature dependences in three temperature intervals separated by T_1 and T_2. For $T<T_1$ the inelastic mechanism is dominant and, as a result, $d_s\simeq d_s^{(B)}\propto T^{-2}$. In the interval $T_1<T<T_2$, the inelastic contribution becomes smaller than the displacement contribution leading to a temperature independent magnetoresistance $d_s\simeq d_s^{(A)}\simeq\mathrm{const}(T)$. Finally, for $T>T_2$ all the contributions are suppressed exponentially [see Eq. (41) and Fig. 2].

Clearly, the T-independent interval does not exist when T_2 is of order or smaller than T_1. This situation was probably realized in the recent experiment.[32] The photoresponse is then expected to cross over from the inverse quadratic dependence $d_s\propto T^{-2}$ for $T<T_2$ to a much faster exponential decay for $T>T_2$. In this case, it is difficult to separate the displacement and inelastic contributions using the temperature dependence of the MIRO only since in the region where the two contributions have comparable magnitudes, their temperature dependence is almost identical [see Fig. 2(b)]. Therefore, for $T_1\gtrsim T_2$ some additional measurements aimed at determination of, for instance, the anisotropic diagonal component d_a of the photoconductivity [Eq. (20)] are desirable. Since d_a [Eq. (21f)] is produced solely by the displacement mechanism (the photovoltaic diagonal contributions $d_a^{(D)}$ and $d_s^{(D)}$ are negligible for $\tau_{tr}/\tau_{sh}\sim 1$), such a measurement would be a reliable tool to estimate the significance of the displacement mechanism at a given temperature. Alternatively, for a fixed polarization of the microwaves, one can analyze the photoresponse in the vicinity of "odd nodes" $\omega/\omega_c=n+1/2$, where the displacement contribution [Eq. (21b)] is nonzero, $\sin^2(\pi\omega/\omega_c)=1$, while the inelastic contribution $\propto\sin(2\pi\omega/\omega_c)=0$ vanishes.

VI. CONCLUSION

In conclusion, we studied the microwave-induced magneto-oscillations (MIROs) in the conductivity of a 2D electron gas with mixed disorder containing both a long-range random potential and short-range scatterers. We calculated all contributions to the current that are linear in the applied dc electric field and quadratic in the microwave field. It is found that the relation between different contributions strongly depends on the strength of wide-angle scattering off disorder and the sample temperature. In general, the microwave radiation modifies both the diagonal and off-diagonal components of the conductivity tensor, including the Hall component. Study of anisotropy of the current may identify each contribution separately. The dominant contribution at low temperatures is the inelastic contribution, which is always diagonal and isotropic. At higher temperatures and in the presence of wide-angle scattering, the displacement mechanism may become larger than the inelastic contribution. At high temperatures the quantum corrections to the conductivity are exponentially suppressed by the electron-electron interaction.

We believe that the possibility of measuring two types of nonequilibrium magneto-oscillations—the MIRO and the HIRO—in the same sample has opened avenues for the characterization of 2D electron systems. In particular, measuring both of them could unambiguously reveal the two-scale structure of disorder characteristic of high-mobility samples. Indeed, the relative weights of the smooth component of disorder (produced by remote impurities separated from the electron gas by a wide spacer) and the short-range component (produced by residual impurities sitting near or at the interface) manifest themselves differently in the various re-

laxation times that could be extracted from the experimental data. As we have demonstrated in this paper, the MIROs are sensitive to the correlation properties of disorder. Specifically, their amplitude and shape are parametrized by the inelastic relaxation time and the disorder-induced relaxation times for the zeroth, first, and second angular harmonics of the electron distribution function, with the displacement contribution to the MIRO—strongly enhanced in the presence of short-range disorder—being parametrized by the time τ_*. Therefore, in addition to the routinely measured τ_{SdHO} (Ref. 73) and τ_{tr}, one can (i) extract the time τ_{q} from the exponential dependence of the amplitude of the oscillations on B, both the MIRO and the HIRO, and (ii) extract the time τ_* from the T and ω dependences of the MIRO, as well as from the amplitude of the HIRO. As for the inelastic time, it can be extracted in a variety of ways: from the power-law T dependence of the MIRO amplitude at lower T, from the MIRO and/or HIRO exponential behavior as a function of T at higher T, as well as from the behavior of the nonlinear dc resistance as a function of the applied dc electric field.[68] Importantly, the inelastic and displacement contributions to the MIRO can then be unambiguously separated from each other. One interesting possibility worthy of experimental investigation is to extract all the relevant parameters in the absence of microwaves and use them to describe the experimental data on the MIRO without any free parameters.

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